

Paper Reference 9FM0/01

Pearson Edexcel

Level 3 GCE

Further Mathematics

Advanced

PAPER 1: Core Pure Mathematics 1

Tuesday 20 May 2025 – Afternoon

Time: 1 hour 30 minutes

Question Paper

**THIS DOCUMENT MUST BE RETURNED TO
PEARSON AT THE END OF THE EXAMINATION.**

In the boxes below, write your name, centre number and candidate number.

Surname					
Other names					
Centre Number					
Candidate Number					

YOU MUST HAVE

**Mathematical Formulae and Statistics Tables (Green),
calculator**

YOU WILL BE GIVEN

Diagram Booklet

Answer Booklet

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or in the separate Diagram Booklet – there may be more space than you need.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **10** questions in this Question Paper.

The total mark for this paper is **75**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1.

$$\mathbf{M} = \begin{pmatrix} 3 & 6 & 0 \\ a & 3 & 1 \\ 2 & a & a \end{pmatrix} \text{ where } a \text{ is a constant}$$

- (a) Determine the values of a for which the matrix $\mathbf{M}$ is singular.
(3 marks)

Given that matrix $\mathbf{M}$ is non-singular and that

- $\det(\mathbf{M}) = -108$
- $a < 0$

- (b) determine the matrix $\mathbf{M}^{-1}$
(3 marks)

(Total for Question 1 is 6 marks)

2. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Determine the exact values of x for which

$$\sinh 2x = 3 \sinh x$$

(Total for Question 2 is 5 marks)

3. The complex number $z = a + bi$ where a and b are real constants.

Given that $\frac{z}{z^*}$ is purely imaginary,

- (a) show that $a^2 = b^2$
(2 marks)

Given also that $zz^* = 50$

- (b) determine the possible complex numbers z
(3 marks)

(Total for Question 3 is 5 marks)

4. (a) Determine the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 2e^{3x}$$

giving your answer in the form

$$y = f(x)$$

(5 marks)

Given that

$$y = 5 \text{ and } \frac{dy}{dx} = 12 \text{ when } x = 0$$

- (b) determine the particular solution of the differential equation.

(3 marks)

(Total for Question 4 is 8 marks)

5. Use the method of differences to prove that for $n > 2$

$$\sum_{r=2}^n \frac{4}{r^2 - 1} = \frac{(pn + q)(n - 1)}{n(n + 1)}$$

where p and q are constants to be determined.

(Total for Question 5 is 5 marks)

6. $f(z) = z^3 + az^2 + bz + c$ where a , b and c are real constants

The roots of the equation $f(z) = 0$ are z_1 , z_2 and z_3

When plotted on an Argand diagram, the points representing these roots form the vertices of a triangle.

Given that

- $z_1 = 2 + 4i$
- the area of the triangle is 12

Determine the two possible functions $f(z)$

(Total for Question 6 is 5 marks)

7. (a) Express

$$\frac{2x^3 + 10x^2 + 9x + 22}{(x + 2)(x^2 + 3)}$$

in partial fractions.

(4 marks)

(b) Hence, show that

$$\int_0^1 \frac{2x^3 + 10x^2 + 9x + 22}{(x + 2)(x^2 + 3)} dx = 2 + \ln\left(\frac{27}{4}\right) - \frac{\pi}{6\sqrt{3}}$$

(4 marks)

(Total for Question 7 is 8 marks)

8. A rambler starts a walk at the bottom of a hill at 10 am

The rambler walks all the way to the top of the hill and then turns around and walks back down to the bottom of the hill.

The differential equation

$$\sin t \frac{dx}{dt} - x \cos t = A \sin 2t \sin t \quad t \geq 0$$

where A is a constant, is used to model the vertical displacement, x metres, of the rambler from the bottom of the hill, t hours after the start of the walk.

Given that after 1 hour

- the rambler has a vertical displacement of 213 metres
- $\frac{dx}{dt} = 273 \cdot 2$

(continued on the next page)

8. continued.

(a) determine the value of A to 3 significant figures.

(1 mark)

(b) Hence, determine the particular solution of the differential equation, giving your answer in the form

$$\mathbf{x = f(t)}$$

(5 marks)

(c) Use the model to determine the time at which the rambler will return to the bottom of the hill.

(3 marks)

(continued on the next page)

8. continued.

Given that the vertical displacement of the top of the hill is 300.68 metres.

(d) use the model to find the value of t when the rambler reaches the top of the hill.

(2 marks)

(e) Using your answers to parts (c) and (d), give a limitation of the model.

(1 mark)

(Total for Question 8 is 12 marks)

9. (i) The curves with equations

$$y = \frac{3}{4}\sinh x \text{ and } y = \tanh x + \frac{1}{5}$$

intersect at just one point **P**

- (a) Use algebra to show that the **x** coordinate of **P** satisfies the equation

$$15e^{4x} - 48e^{3x} + 32e^x - 15 = 0$$

(3 marks)

- (b) Show that $e^x = 3$ is a solution of this equation.

(1 mark)

- (c) Hence state the exact coordinates of **P**
(1 mark)

(continued on the next page)

9. continued.

(ii) Show that

$$\int_{-4}^0 \frac{e^{\frac{1}{x}}}{x^2} dx = e^{-\frac{1}{4}}$$

(4 marks)

(Total for Question 9 is 9 marks)

10. (a) Given that, for $n \in \mathbb{N}$

$$z^n + \frac{1}{z^n} = 2 \cos n\theta$$

$$z^n - \frac{1}{z^n} = 2i \sin n\theta$$

show that

$$8 \sin^4 \theta \equiv \cos 4\theta - 4 \cos 2\theta + 3$$

(5 marks)

(continued on the next page)

10. continued.

Refer to Diagram 1 and Diagram 2 for Question 10(b) in the Diagram Booklet.

Diagram 1 shows the central vertical cross-section of a solid wooden ornament.

Diagram 2 shows the curve with equation

$$x = \sin^2\left(\frac{1}{2}y\right) \quad 0 \leq y \leq \frac{8\pi}{5}$$

The region **R**, shown shaded in Diagram 2, is bounded by the curve, the line with equation $y = \frac{8\pi}{5}$ and the **y**-axis.

The ornament is modelled by the solid of revolution formed when **R** is rotated 360° about the **y**-axis.
The units are centimetres.

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10. continued.

- (b) Using algebraic integration and the result in part (a), determine, in cm^3 , the volume of wood needed to make the ornament, according to the model.

Give your answer to **2** significant figures.

[Solutions based entirely on calculator technology are not acceptable.]

(5 marks)

Given that

- the density of the wood is 0.85 g/cm^3
- the mass of the ornament is **6** grams

- (c) comment on the suitability of the model.

(2 marks)

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER